Common Distributions and Moment Generating Functions

1 Discrete Distributions

distribution	pmf $p(k)$	argument	MGF : $M(\theta)$	MGF domain
Binomial(n, p)	$\binom{n}{k} p^k (1-p)^{n-k}$	$k = 0, 1, \ldots, n$	$\left(1 - p + e^\theta p\right)^n$	$\theta \in R$
Geometric(p)	$p(1-p)^{k-1}$	$k = 1, 2, 3, \ldots$	$\frac{e^\theta p}{1-(1-p)e^\theta}$	$e^\theta < \frac{1}{1-p}$
Negative Binomial(r, p)	$\binom{k-1}{r-1} p^r (1-p)^{k-r}$	$k = r, r+1, \ldots$	$\left(\frac{e^\theta p}{1-(1-p)e^\theta}\right)^r$	$e^\theta < \frac{1}{1-p}$
Poisson, λ	$\frac{\lambda^k}{k!} e^{-\lambda}$	$k = 0, 1, 2, \ldots$	$e^{\lambda(e^\theta - 1)}$	$\theta \in R$

Notice that the geometric is a special case of the negative binomial distribution.

2 Continuous Distributions

distribution	pdf $f(x)$	support	MGF : $M(\theta)$	MGF domain
$N(\mu, \sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$	$x \in R$	$\exp\left\{\mu\theta + \frac{1}{2}\sigma^2\theta^2\right\}$	$\theta \in R$
$\Gamma(\alpha, \lambda)$	$\frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$	$x > 0$	$\left(\frac{\lambda}{\lambda - \theta}\right)^\alpha$	$\theta < \lambda$
Uniform$(0, 1)$	1	$x \in [0, 1]$	$\frac{e^\theta - 1}{\theta}$	$\theta \in R$

Special Cases :

1. The exponential, λ distribution is the $\Gamma(1, \lambda)$ distribution.
2. The $\chi^2_{(n)}$ distribution is the $\Gamma\left(\frac{n}{2}, \frac{1}{2}\right)$ distribution.