University of Western Ontario
Statistics 3657 Term Test I

October 16, 2013, 5:30 PM - 6:20 PM

Instructor: R. J. Kulperger

1	
2	
3	
4	
Total (50)	

Name : ______________________________

ID : ____________________

Instructions:

I. Make sure that your name and ID number are the front page, and every page, of your question booklet.

II. This term test is of 50 minutes duration. Each question starts on a new page, and space is given for the answer.

Each question is worth approximately the same marks.

III. All questions are to be answered on the questions sheets. You may use the back of these pages, but please indicate where any answer continuation is located. In the absence of such directions work on the backs of pages will not be graded.

IV. An additional booklet will be provided, if needed, for your rough work. This will not be graded.

1. **This problem continues on the next page**

   (a) State the three axioms of probability.

   (b) Using these axioms prove the following : If A, B are two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

   (c) Suppose that X, Y and Z are i.i.d. geometric parameter θ distribed r.v.s. Show that $V = X + Y$ and $W = 2^Z$ are independent.

   Hint : You may do this directly or by stating an appropriate theorem and applying it.

2. **This problem continues on the next page**

   (a) Suppose that X has a pdf

   $$f_X(x) = cxe^{-x} I(x > 0) \ .$$

   Find the constant c so that this is a pdf.

   Find the cdf of X.

   Let $Z = e^X$. Find the cdf and then the pdf of Z.

   (b) Suppose X and Y have bivariate pdf f. Let $T = X + Y$. Using the cdf of T *derive* the formula for the pdf of T in terms of the pdf's f the joint pdf of X, Y.

(c) Suppose $X \sim \text{Unif}(0, 1)$ and $Y \sim \text{Unif}(-1, 0)$ are independent r.v.s. Let $T = X + Y$. Calculate the pdf of T. Sketch the pdf of T.

3. **This problem continues on the next page**

   (a) Suppose that F is a continuous cdf (on the reals). Suppose that $X \sim F$. State an appropriate theorem or proposition, that is conditions and conclusions, that gives the distribution of $F(X)$. **Do Not Prove This Theorem**.

   (b) Consider the pdf

$$f(x) = \begin{cases} \frac{2x}{9} & \text{if } 0 \le x \le 3 \\ 0 & \text{otherwise .} \end{cases}$$

   Suppose that $U \sim \text{Uniform}(0, 1)$.

   State a theorem or proposition, including conditions, that gives a function h so that $X = h(U)$ has the distribution with pdf f above. Prove this theorem.

(c) Give this function h so that $X = h(U)$ has the distribution with pdf f from the previous part.

4. (a) Suppose X has pdf

$$f_X(x) = \frac{1}{2}x^2 e^{-x} I(0 < x < \infty) \ .$$

The conditional pdf of Y given that $X = x$ is

$$f_{Y|X=x}(y) = \frac{2y}{x^2} I_{(0,x)}(y) \ .$$

Based on this information determine if X and Y are independent, and justify your answer.

(b) Find the joint probability density function (pdf) of X, Y.

(c) Find the marginal pdf of Y.