1. The r.v.s N and $X_i, i \geq 1$ are all independent, and the X_i are iid Poisson, 1, and N has pmf

$$P_N(n) = P(N = n) = \begin{cases} \frac{1}{4} & \text{if } n = 1 \\ \frac{1}{2} & \text{if } n = 2 \\ \frac{1}{4} & \text{if } n = 3 \\ 0 & \text{otherwise.} \end{cases}$$

Consider the r.v. (the random sum of r.v.s)

$$S = \sum_{i=1}^{N} X_i \ .$$

(a) Obtain (derive) the formula for $\mathrm{E}(S|N = n)$ and $\mathrm{E}(S^2|N = n)$.

(b) Give the r.v.s $E(S|N)$, $E(S^2|N)$.

(c) State an appropriate Theorem and use it to find $\mathrm{E}(S)$, $\mathrm{E}(S^2)$ and $\mathrm{Var}(S)$.

(d) Find $P(S = 1)$. *Hint : Use conditional probability. You may also use the property that a sum of* m *iid Poisson,* λ*, r.v.s has a Poisson,* $m\lambda$*, distribution.*