SOME HELPFUL FORMULAE

1. Gamma function and properties:
   (a)
   $$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$$
   (b) For $\alpha > 0$, $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$.
   (c) For integers $n \geq 1$, $\Gamma(n) = (n-1)!$.
   (d) $\Gamma(1/2) = \sqrt{\pi}$
2. Binomial, parameter θ and n the number of trials
   $$p(k) = \binom{n}{k}(1-\theta)^{n-k}\theta^k \ , k = 0, 1, 2, \ldots, n \ .$$
3. Discrete uniform on $A = \{1, 2, \ldots, m\}$
   $$p(k) = \frac{1}{m} \ , \ k \in A \ , \ \text{and } p(k) = 0 \text{ otherwise.}$$
4. The Poisson distribution with parameter λ has pmf
   $$p(k) = \frac{\lambda^k e^{-\lambda}}{k!} \ , \ k = 0, 1, 2, \ldots \ .$$
5. Geomteric(p) : $X \sim$ Geometric(p) where X is the number of failures before the first success in iid Bernoulli trials. Similarly Y = number of trials to first success is also said to have a geometric distribution. Here $Y = X + 1$.
   $$P(Y = k+1) = P(X = k) = p(1-p)^k \ , \ k = 0, 1, 2, \ldots$$
   and $P(X = x) = 0$ for all x not non-negative integers.
6. Uniform on the interval (a, b), where $a < b$
   $$f(x) = \frac{1}{b-a} I(a < x < b).$$
7. Gamma(α, β) pdf:
   $$f(x) = \begin{cases} \frac{\beta^\alpha x^{\alpha-1}}{\Gamma(\alpha)} e^{-\beta x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$
8. chi-square distribution with degrees of freedom k : Gamma($\frac{k}{2}, \frac{1}{2}$)
9. The Beta(α, β) pdf is
   $$f(x) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

10. The bivariate normal pdf is

$$\begin{aligned} f(x,y) &= \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_X)^2}{\sigma_X^2}\right.\right. \\ &\quad \left.\left. -2\rho\frac{(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y} + \frac{(y-\mu_Y)^2}{\sigma_Y^2}\right]\right\} \end{aligned}$$

11. (a) $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

    (b) $\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x$

12. $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$