Urn Game Binary Tree Plots

Choose balls from an urn. The urn has $R = 4$ red balls and $W = 6$ white balls. Figure 1 shows the probability rules for a game **without** replacement and Figure 2 shows the probability rules for a game **with** replacement.

Consider the events A_i of drawing a red ball on draw i. For each game show that $P(A_i) = \frac{4}{4+6} = \frac{4}{10}$.

Show that for the the **without** replacement game

$$P(A_1 \cap A_2) \neq P(A_1)P(A_2)$$

and so A_1 and A_2 are dependent.

More generally if there are R red and W white balls then

$$\begin{aligned}
P(\text{red on draw 2}) &= P(R1R2) + P(W1R2) \\
&= P(R2|R1)P(R1) + P(R2|W1)P(W1) \\
&= \frac{R-1}{R+W-1} \times \frac{R}{R+W} + \frac{R}{R+W-1} \times \frac{W}{R+W} \\
&= \frac{R}{(R+W)(R+W-1)} \times (R-1+W) \\
&= \frac{R}{R+W}
\end{aligned}$$

Here the notation $R1$ and $R2$ are short hand for red on draw 1 and red on draw 2. The notation $W1$ and $W2$ similarly refer to white on draws 1 and 2.

Show that for the the **with** replacement game

$$P(A_1 \cap A_2) = P(A_1)P(A_2)$$

and so A_1 and A_2 are independent.

Are A_1 and A_3 independent for either of these games?

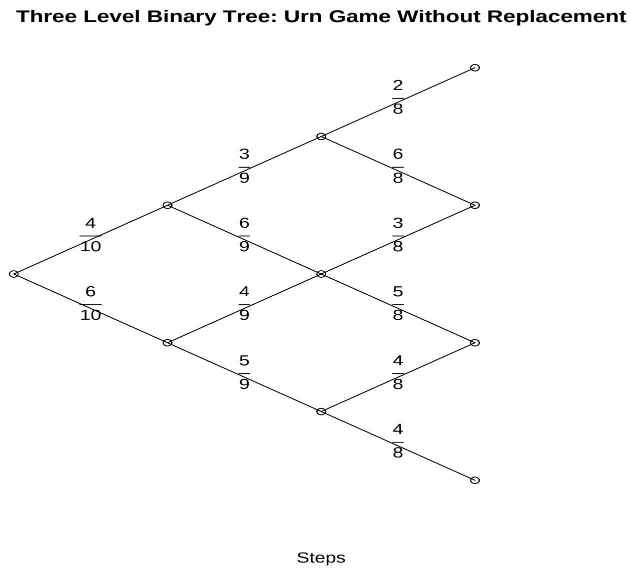

Figure 1: Choose From an Urn Without Replacement

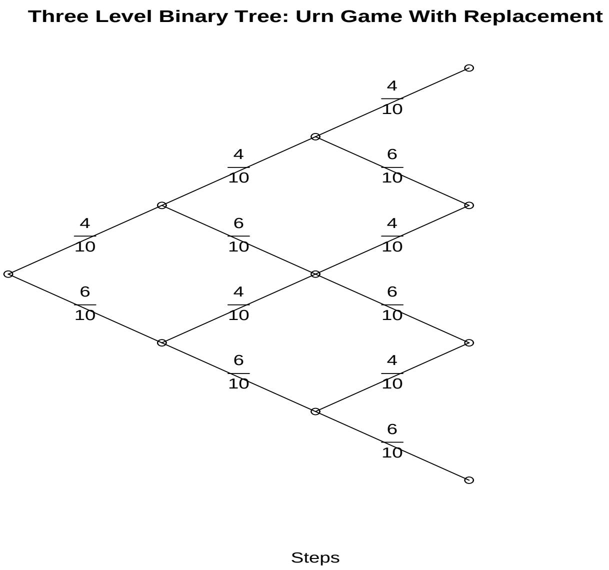

Figure 2: Choose From an Urn With Replacement