Statistics 3657a Assignment 4

Handout date: 14 November, 2014 Due date: 28 November, 2014

Problems from the text unless otherwise noted. The third bullet is a problem not from the text. The assignments are due at the beginning of class on the due date.

• Section 4.7 : 13, 31, 44, 53 (also $-\sqrt{\operatorname{Var}(X)\operatorname{Var}Y} \leq \operatorname{Cov}(X, Y)$, but the problem does not ask you to show this), 59, 77, 80, 103

Many of these problems have short answers.

• Section 5.4 : 1, 4, 16, 21 (a) - (b)

Many of these problems have short answers.

• Chapter 5 : In the same setting as Theorem B, consider the r.v.

$$Y_n = \frac{1}{\sqrt{n}} S_n \; .$$

Aside : there is no σ in the denominator.

Let M_n be the moment generating function of Y_n .

Show that

$$\log(M_n(t)) \to \frac{\sigma^2 t^2}{2}$$

as $n \to \infty$.

Hint : See the notes for our proof of the CLT and follow this idea. However in this case $M''(0) = E(X^2) = \sigma^2$.

What is the limit distribution of Y_n ? Justify this by using the continuity theorem.

Notes on some of the questions :

- 4.7.13 is a relation that is used in some areas of actuarial science.
- 4.7.54 (suggested problem): The r.v.s U, V have a common random factor Z. This induces a dependence between U, V, even though conditional on Z, the random variables U, V are conditionally independent giv en Z = z. This idea is exploited in economics in factor models, in actuarial science and finance in frailty models, random effects models such as stochastic volatility models and hidden Markov models, random effects models in biostatistics and design of experiments.
- 5.4.21 (not on the assignment) brings an idea that one can use distributions other than uniform in order to use a simulation (or Monte Carlo) method to approximate the value of an integral. Students in financial mathematics will see several methods of how to perform simulations, including this is one which is the basis of the so called *importance sampling*.

Below are problems from the text, but not part of the assignment. These are some suggested problems that are helpful in preparing for the exam in December.

Section 4.7: 15, 33, 34, 43, 45, 47, 48, 49, 54, 57, 60, 62, 63, 68, 69, 77, 81, 83, 89, 95, 99, 100, 101, 102