

# Statistics 3858b Assignment 5

Handout March 30, 2016 ; Due date: April 7, 2016

This assignment is optional. These are some problems dealing with Bayesian calculations.

These problems are all from the course text unless otherwise stated.

1. 8.10.4 Use this distribution and the prior as in (e) and the conditional distribution of  $X$  as given in the beginning of this problem. Consider  $n = 1$  observation  $X_1$ . Calculate the posterior mean, that is the Bayes estimator.

Now do this with  $n = 2$ . Suppose the observed data is  $x_1 = 3, x_2 = 0$ . Obtain the posterior distribution and plot it. On this same plot also plot the prior.

2. 8.7.10 (d).
3. Consider the Bayesian problem with  $X|\Lambda = \lambda = \lambda \sim \text{Poisson}, \lambda$  and with prior  $\Lambda \sim \text{Gamma}(\alpha, \beta)$ .

Suppose we observe, conditionally on  $\Lambda$   $n$  iid r.v.s  $X_1, \dots, X_n$ .

[You may wish to review the class notes as this is one of the examples we studied.]

- (a) Is this prior a conjugate prior? Your answer will be yes or no, and with a sentence or 2 to justify it. Your answer or course will be related to the next part where you calculate the posterior distribution.
- (b) Give the posterior distribution of  $\Lambda$
- (c) Give the Bayes estimator.
- (d) Suppose  $\alpha = 2, \beta = 1, n = 2$  and  $x_1 = 2.1, x_2 = 3.9$ . Give the posterior distribution, the posterior mean.

Also give the posterior 0.95 prediction interval. This is the interval obtained from the .025 and .975 quantiles of the posterior distribution. You can calculate this using the appropriate function in R for Gamma distributions.

4. Consider a r.v.  $X$  with mean  $\mu$  and variance  $\sigma^2$ . Consider the function  $g$  with argument  $a \in \mathcal{R}$  (=the real numbers) given by

$$g(a) = E((X - a)^2) .$$

Show that  $g$  is minimized when  $a = \mu$ .

*Aside* : It is based this type of argument and conditional expectation that one defines the Bayes estimator as the mean or expectation of the posterior distribution.

**Notice** The final exam is on April 10, 7 PM. See the course web page for the room.